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Vortex mutual friction in superfluid ^{3}He

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Abstract. After a historical introduction to the discovery of vortex mutual friction in He II, the equivalent modern experiment on superfluid ^{3}He is described and its interpretation discussed.

1. Introduction: a little history

I have chosen to talk about experiments carried out at Manchester over the last few years because they are the direct descendant of a key experiment [1] that Joe Vinen and I did when we were both research students at the Royal Society Mond Laboratory in Cambridge. This experiment was Joe's idea, and I got in on the act because I had a cryostat in which things could be rotated. I would like to suggest that, despite Joe's many good ideas since then, this remains his most profoundly original idea, because he conceived the experiment prior to the notion of quantized vortices.

It is difficult now to cast one's mind back to the days before quantized vortices were invented. A flavour is perhaps given by a short note [2] that we wrote, at Brian Pippard's instigation, shortly before reading Feynman's [3] astonishing article, which dissolved the mists surrounding Onsager's [4] earlier Delphic utterances.

Joe had been given the task of investigating the paradox that mutual friction appeared in a steady heat current but not in the alternating counterflow of second sound. Very fortunately he decided to study the problem in the time domain rather than the frequency domain—a crucial decision for a non-linear phenomenon. He found that the propensity for mutual friction appeared only after a time delay when a heat current was switched on, and decayed slowly after it was switched off. Once the propensity was established, mutual friction was manifest both as a longitudinal temperature gradient in a heat current and as second-sound attenuation [5]. This time behaviour was very reminiscent of the growth and decay of turbulence in an ordinary liquid. But turbulence in superfluid helium would require violation of the Landau condition curl $\boldsymbol{v}_s = 0$. Although many people did not take this condition very seriously at the time, Joe was led to conjecture that mutual friction might be a property of any non-irrotational flow. The simplest known such flow was macroscopically uniform rotation [6], and Joe therefore suggested that we should look for second-sound attenuation in uniformly rotating helium. The surprising nature of this prediction is emphasized by the fact that when Brian Pippard found out what we were doing he hazarded the guess that there would be no effect.

One further recollection of that period will surprise no one who has subsequently worked with Joe. When we came to work out the theory of vortex mutual friction [7], I was most impressed by the fact that Joe had not only read and understood Schiff's '*Quantum Mechanics*', but was able to calculate the Born approximation (of which I had not then heard) for the non-trivial $\boldsymbol{p} \cdot \boldsymbol{v}_s$ interaction.

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2. The experiment

Since the Manchester experiments on vortex mutual friction in superfluid ^{3}He have already been fully reported [8], I shall in this article confine myself to a qualitative exposition of the experiments and major features of their interpretation. For measurements of mutual friction the major difference between ^{3}He and ^{4}He is that the viscosity of ^{3}He is very much larger and the entropy is very much smaller. This has the consequence that second sound essentially does not exist as a propagating mode in ^{3}He, and so cannot be used to measure mutual friction.

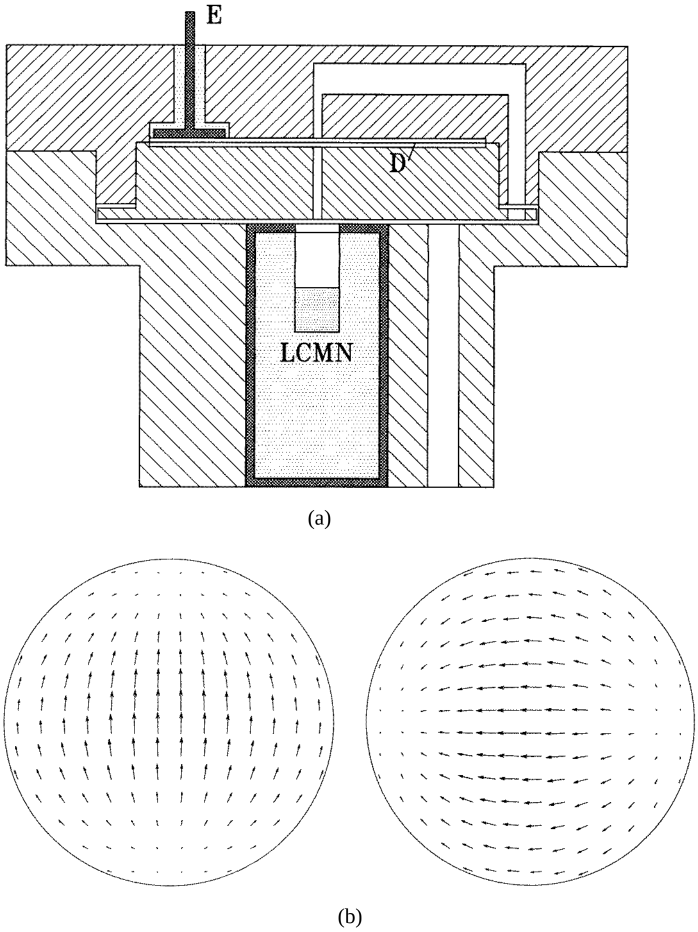


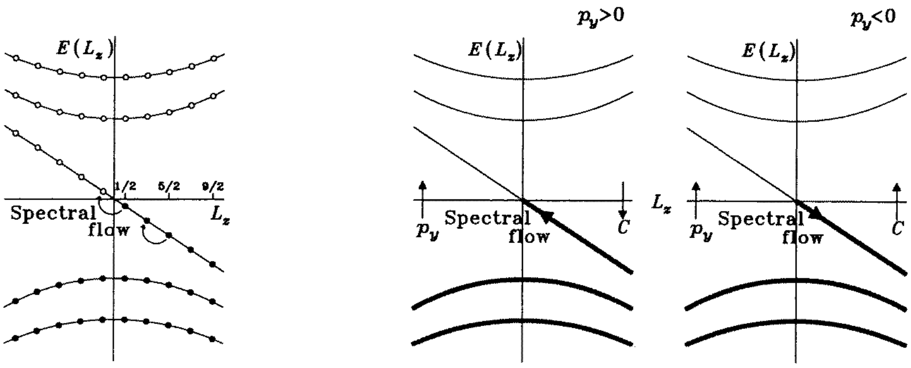
Figure 1. (a) A simplified cross section of the experimental cell, bolts and O-rings not shown. The aluminized Kapton diaphragm D drives superflow in the thin layers of liquid (magnified about 5× in thickness) above and below it. Motion of the diaphragm is driven and detected by six electrodes in the roof of the cell, one of which is shown at E. Below the cell the LCMN thermometer pill is shown, enclosed with its coil assembly in a niobium can. The whole unit plugs into the top of the nuclear refrigeration stage. Note the liquid-filled channels through the copper pieces (diagonally shaded) by which liquid above and below the diaphragm makes thermal contact with the LCMN thermometer and with the nuclear stage. (b) Superfluid velocity fields for the two orthogonal diaphragm modes used in most of this work. The non-rotating bandwidth of these modes is smaller than that of higher-frequency modes, so they yield the most accurate mutual friction parameters. Within experimental error the measured mutual friction was the same for all modes.

Instead we make use of the high viscosity to lock the normal fluid and study an oscillating mode related to fourth sound. But instead of using the compressibility of the liquid to provide the restoring force we use the higher compliance associated with the displacement of a plastic diaphragm. This has the effect of reducing the resonant frequency below about 50 Hz, so the normal fluid is more fully locked.

The essential features of our experimental cell are shown in figure 1(a). A circular Kapton diaphragm separates two layers of liquid 40 mm in diameter and 100 μm thick. In the mode where one half of the diaphragm moves up while the other moves down, superflow is driven as shown in figure 1(b), in opposite directions on the two sides of the diaphragm. The two normal modes shown are ideally degenerate, but in our apparatus differ in frequency by 3.2%, probably because of anisotropic diaphragm tension. The mutual friction due to uniform rotation with angular velocity Ω can be resolved into forces parallel and perpendicular to the superflow, specified respectively by resistive and reactive coefficients α and α' [9]. The resistive force produces a broadening of the diaphragm resonance proportional to $\alpha\Omega$, and the reactive force produces a coupling between the two modes shown in figure 1(b) proportional to $(1-\alpha')\Omega$, in which the 1 is the effect of the Coriolis force.

3. Spectral flow

It turns out that in ^{3}He-B the dominant scattering of free excitations is not by the $\boldsymbol{p}\cdot\boldsymbol{v}_s$ interaction, but by the excitations bound to the vortex core that were discovered in superconductors by Caroli, de Gennes and Matricon [10]. A crucial feature of the spectrum of bound excitations shown in figure 2(a) is the existence of an asymmetric branch, which means that, under a suitable perturbation, quasiparticles can be created without the simultaneous production of quasiholes. Just such a perturbation is provided when the vortex moves relative to the normal fluid. Scattering between bound and free excitations tends to equate their drift velocities, with the result that the bound excitations move relative to the vortex. This results in a continuous



(a) (b)

Figure 2. (a) The spectrum of bound states on a vortex core; filled circles show occupied states and open circles empty states. Spectral flow along the asymmetric branch, induced by normal flow past the vortex, results in a continuous change of angular momentum. (b) Motion of the bound excitations in the x-direction relative to the vortex leads to an increase or decrease of angular momentum depending on the sign of p_y. Consequently excitation momentum p_y is continually created from the superfluid ground state, although chiral charge C is conserved because quasiparticles and quasiholes are created in equal numbers.

increase or decrease of angular momentum, depending on the direction of the linear momentum of the excitation, as indicated in figure 2(b). The net result is a continuous generation of excitation momentum from the superfluid background in a direction perpendicular to the drift motion of the bound excitations relative to the vortex, and hence a mutual force between normal fluid and superfluid. Stone [11] has given a particularly illuminating discussion of this spectral flow phenomenon by analysing the time evolution of bound-state wavefunctions.

Typical experimental results for the B phase are compared with the spectral flow model in figures 3 and 4. The ratio of mode coupling to dissipation shown in figure 3 depends only on the relaxation parameter governing the coupling between bound and free excitations, whereas the mode coupling itself, shown in figure 4, is independent of relaxation. Both figures show a good qualitative fit to the model, which becomes almost quantitative if an energy gap less than the theoretical one is used. This has led us to speculate that spectral flow may be assisted by a phenomenon analogous to a mobility edge below the gap.

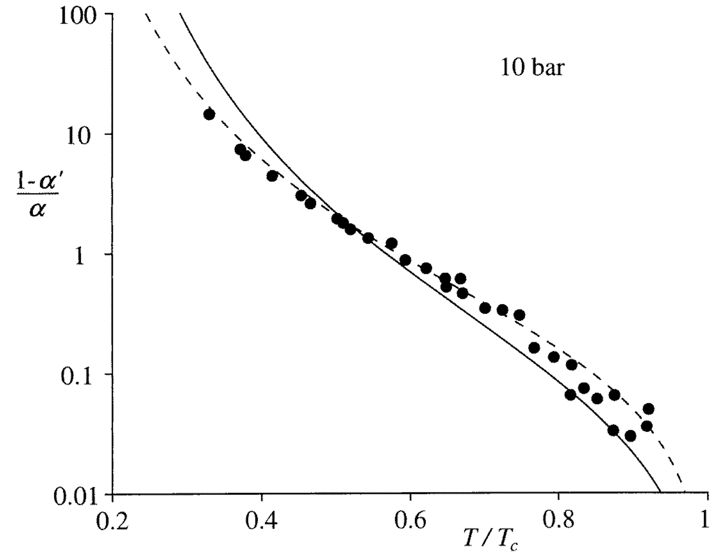


Figure 3. Experimental values of the ratio of mode coupling to dissipation, $(1-\alpha')/\alpha$, at 10 bar compared with the theoretical relaxation parameter $\omega_0\tau$. The curves have one fitting parameter for the overall scale. The full curve is for the theoretical energy gap $\Delta(T)$, and the broken curve is for a reduced effective gap $\Delta_{\rm eff}(T) = 0.62\Delta_{\rm bulk}(T)$.

4. Textural dynamics

In the A phase, continuous vortices can be formed as a texture of the $\hat{l}$-vector, which is the direction of the orbital angular momentum of the Cooper pairs. Rotation of the order parameter about $\hat{l}$ is equivalent to a phase change, and it is the consequent coupling of flow to texture that produces distributed vorticity. A simple example of a continuous vortex texture is shown in figure 5. Motion of this type of vortex involves local reorientation of the $\hat{l}$-vector, and is thus governed in the hydrodynamic regime by the equations of orbital hydrodynamics. The relevant macroscopic coefficients are the orbital inertia, representing the gyroscopic response of $\hat{l}$, and the orbital viscosity, representing dissipation associated with the motion of $\hat{l}$. According to Volovik [12] a spectral flow effect near the gap nodes reduces the orbital

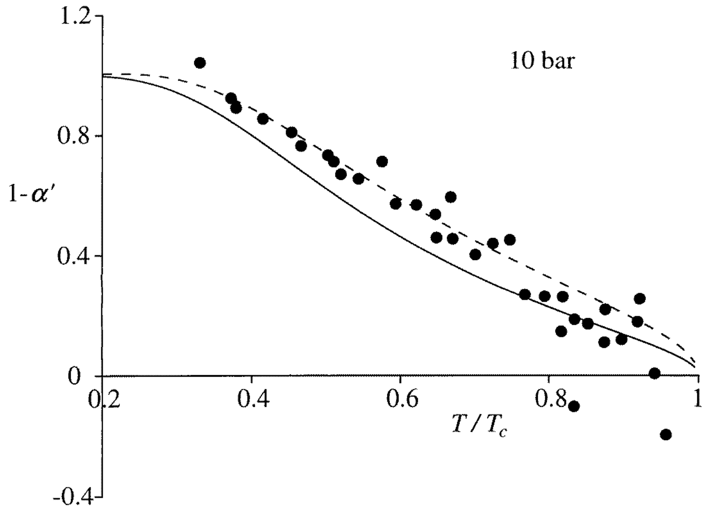


Figure 4. Experimental values of the mode coupling parameter $(1-\alpha')$ at 10 bar compared with the prediction of the spectral flow theory, which is independent of the relaxation parameter $\omega_0\tau$. The full curve is for the theoretical value of $\Delta(T)$ and the broken curve is for $\Delta_{\rm eff}(T) = 0.62\Delta_{\rm bulk}(T)$.

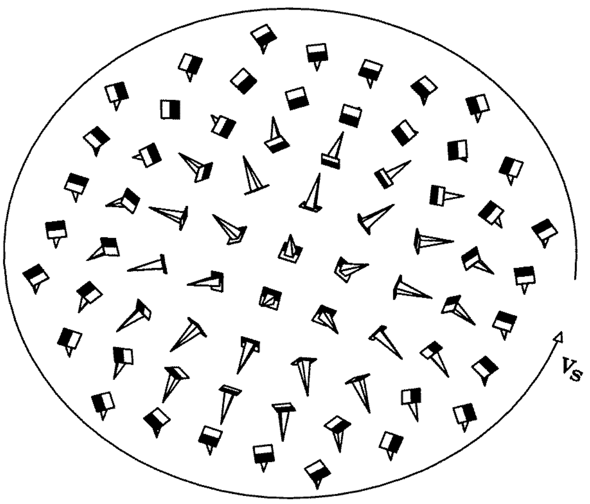


Figure 5. An example of a continuous vortex in ^{3}He-A. The cones indicate the local direction of the order parameter vector $\hat{l}$ which is parallel to the angular momentum of the Cooper pairs. Rotation of the cones about $\hat{l}$ indicates a change in phase of the order parameter; note the 4π phase change around the perimeter of the diagram, corresponding to two quanta of anticlockwise circulation.

inertia by several orders of magnitude from a naive value of order $\hbar$ per Cooper pair; this is unmeasurably small.

Comparison of theory with experiment is most conveniently done via parameters $d_\parallel$ and $d_\perp$ that relate the mutual friction to the motion of the normal fluid relative to the vortex in the same way that α and α' relate it to normal motion relative to the superfluid. The orbital viscosity is proportional to $d_\parallel$ and the orbital inertia to $1 - d_\perp$. The comparison is shown in

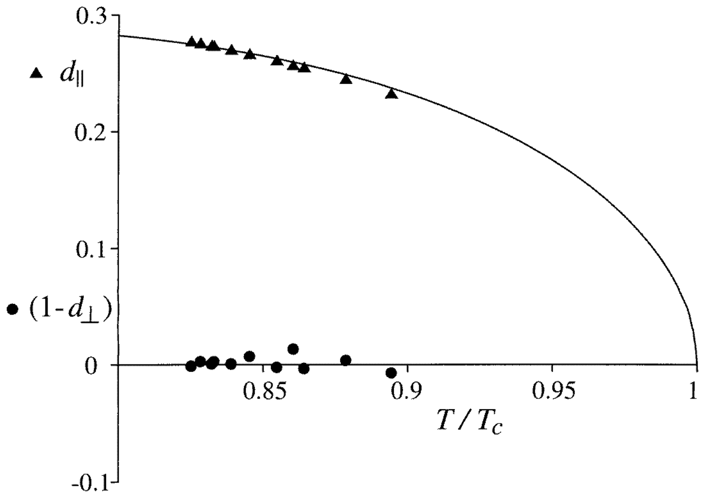


Figure 6. $d_{\|}$ and $(1 - d_{\perp})$ in the A phase at 29.3 bar. The full curve is the theoretical orbital viscosity and the x-axis corresponds to zero orbital inertia.

figure 6. The orbital viscosity agrees in temperature dependence with theory, and the absolute magnitude is close to expectation. The mean orbital inertia per pair is $(0.0015 \pm 0.0017)\hbar$, consistent with Volovik's prediction.

5. Conclusions

^{3}He is distinguished from ^{4}He by the existence of a well established and reasonably successful microscopic theory, so we have been able to reach a fairly complete theoretical understanding of the mutual friction experiments. But I would not wish to leave you with the impression that the interpretation that I have presented is without controversy. Despite the lapse of over three decades since the pioneering work of Nozières and Vinen [13] the precise role of the Magnus effect is still under discussion [14]. Nevertheless, I believe that the interpretation that I have presented here is broadly correct. I also believe that it is important to understand the Galilean-invariant superfluid ^{3}He in order to provide a sound basis for understanding the more complicated case of superconductors.

Finally, it is of some historical interest to compare the present work with the original work on He II [1]. The most noticeable difference is the very much larger number of high-quality data points in the recent work. This is an obvious effect of the use of computers to record and analyse data, and it can reveal effects otherwise hard to spot, such as the vortex core transition in the B phase. The analysis required to get good measurements of mode coupling, and hence the transverse force, would not have been practicable without computers. However, it is perhaps fortunate that these facilities were not available in 1955: had we made detailed measurements near the λ-point we would have been puzzled by results quite at variance with the simple theory that we developed at that time.

One might expect that the use of computers would reduce the manpower required to run an experiment, but of course one is led to attempt more. This, together with the complications of nuclear refrigeration, has led to an increase in manpower. I would therefore like to conclude by acknowledging the crucial contributions of John Hook and the several postdoctoral fellows, research students and technicians who have been involved in this work.

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